ON DAMAGE MODELLING IN UNSATURATED CLAY ROCKS

C. Arson[1], B. Gatmiri[2,1]

1. Univ. Paris-Est , U.R. NAVIER (CERMES)
   Ecole Nationale des Ponts-et-Chaussées
   6-8 av. Blaise Pascal
   Cité Descartes, Champs-sur-Marne
   77455 Marne-la-Vallée cedex 2
   FRANCE
   Tel : 00 33 1 64 15 35 24
   Fax : 00 33 1 64 15 35 62
   arson@cermes.enpc.fr, gatmiri@cermes.enpc.fr
2. University of Tehran, Department of Civil Engineering

Abstract

The aim of this paper is to present the main problems encountered in the modelling of damage in an unsaturated quasi-brittle rock mass. Micromechanical damage models are based on a physical definition of damage, related to fracturing. Phenomenological formulations are less straightforward, but offer huge modelling possibilities by means of economical computation processes. Due to the dissipative aspect of damage, the Inequality of Clausius-Duhem (ICD) has to be satisfied. Strain softening and crack localization are regularized by means of a non-local formulation, founded on microstructure concepts, homogenisation and space averaging or gradient-enhancement. In an unsaturated damaged porous medium, suction effects combine with mechanical loading and fracturing, which induces complex couplings. On the one hand, Continuum Damage Mechanics well represents stiffness degradation for dry materials. On the other hand, fracture network models give a good estimation of complex flows. It is difficult to reconcile both theories A new mixed model, HHMD, is proposed. It is a fully coupled formulation, involving independent state variables.

Key-words

damage, independent state variables, poromechanics, micromechanics,, phenomenological damage model, localization, multimodal flow model

Introduction

This work is part of research dedicated to the study of the Excavation Damaged Zone surrounding galleries for nuclear waste repositories. It is assumed that the material to be modelled is an unsaturated quasi-brittle clay rock.

All of the following statements apply for isothermal conditions. Attention is thus focused on the behaviour of the rock mass before disposal.

Damage modelling involves several theoretical frameworks. The dissipative side of damage involves thermodynamic requirements. Strain softening and crack localization make the problem ill-posed if the damage model is not regularized. Moreover, modelling of the EDZ surrounding deep repositories must take into account saturation variations around the excavation. This involves complex hydro-mechanical couplings, both in the behaviour laws and in the fluid transfer equations.

This work focuses on the main issues to be solved in order to model the hydro-mechanical behaviour of a damaged unsaturated rock mass. A representative sample of existing theoretical frames is reviewed and examined. Then, a new mixed damage model, formulated in independent state variables, is outlined. The first section presents the main keys provided by Continuum Damage Mechanics emphasizing the extension of damage models to unsaturated materials. The second part presents the principles of hydraulic transfer estimation in fracture networks, with emphasis on. the introduction of damage in hydraulic transfer models.

1. Continuum–based mechanical damage concepts

Damage of geomaterials is physically related to the fracturing process. However, damage is not always quantified by crack parameters. In some models, more attention is paid to the representation of damage than to its definition. In these particular cases, what is modelled is stiffness degradation and permeability increase rather than crack opening. Damage is thus often an abstract concept, defined indirectly from its influence on the material behaviour. A parallel can be drawn between damage and plasticity models. Damage also involves dissipative processes. Therefore, mechanical damage theories have to satisfy rules of irreversible thermodynamics. Moreover, localization limiters have to be found out in order to avoid the concentration of irreversible strains. However, models dedicated to unsaturated porous media are less common in damage mechanics than in elastoplastic theory. Most of the existing formulations are based on Biot's effective stress concept, which relies on Bishop's stress definition. This latter choice seems not to be valid for unsaturated behaviour, as shown by Fredlund and Morgenstern (Fredlund and Morgenstern, 1977). One consequence is that it neglects couplings between damage and hydraulics. Another theoretical frame will be proposed.

1.1. **Types of models**

1.1.1. Micromechanical models

Definition and representation of micromechanical damage

The micromechanical approach models the influence of local damage on macro-mechanical behaviour. Damage variables have a physical meaning related to the degradation of elastic properties or to the characteristics of the fracture network. It is assumed that stresses are redistributed due to a decrease of the effective material area.

Stress-strain relationships are thus written in terms of effective variables. The effective stress is the stress that develops in the fictive undamaged counterpart of the system (de Borst et al., 1999). Its definitions require the use of a fourth-order effective-stress operator (Hansen and Schreyer, 1994):

$$\hat{\underline{\underline{\sigma}}} = \underline{\underline{\underline{\underline{M}}}}(\Omega) : \underline{\underline{\sigma}} \tag{1}$$

where the damage variable Ω can be a tensor. The effective stress concept is often combined with the Principle of Equivalent Elastic Energy (PEEE) to compute the damaged rigidity tensor $\underline{\underline{\underline{\underline{D_e}}}}(\underline{\underline{\Omega}})$. As recalled in (Hansen and Schreyer, 1994), this approach postulates that the elastic energy of the intact material subjected to the effective stress $\hat{\underline{\underline{\sigma}}}$ is equal to the elastic energy of the damaged material subjected to the real stress $\underline{\underline{\sigma}}$ $\left(W_e(\hat{\underline{\underline{\sigma}}}, \Omega = 0) = W_e(\underline{\underline{\sigma}}, \Omega)\right)$, which leads to the equality:

$$\underline{\underline{\underline{\underline{D_e}}}}(\Omega) = \underline{\underline{\underline{\underline{M}}}}(\Omega)^{-1} : \underline{\underline{\underline{\underline{D_e^0}}}} : \underline{\underline{\underline{\underline{M}}}}(\Omega)^{-T} \tag{2}$$

The definition of an effective stress provides a framework to determine the damaged mechanical properties of the material. However, damage remains an abstract notion, represented by its influence on behaviour laws. That is why in some models, damage is also given a physical meaning, generally related to fracturing. A common approach consists in gathering cracks into "families" of close orientations (Swoboda and Yang, 1999b; Shao and Rudnicki, 2000; Shao, Ata and Ozanam, 2005). Following the principle of spectral decomposition of Ortiz (Ortiz, 1985), a second-order damage variable is defined as follows:

$$\underline{\underline{\Omega}} = \sum_{i=1}^{3} d_i \underline{n_i} \otimes \underline{n_i} \tag{3}$$

Equality (3) infers that the material is fractured in three principal directions $\underline{n_i}$, whose importance is weighed by the crack densities d_i. Definition (3) assumes that cracks are non-interacting. Bazant (Bazant, 1991) proposed to take fracture interactions into account by defining crack opening as a function of the energy release rates of every crack of the Representative Elementary Volume (REV), weighed by interaction coefficients specific to each pair of fractures.

Non-locality of micromechanical damage theories

Since the micromechanical methods update macro-data using micro-data, the theory is non-local. Bazant gave micromechanical evidences of the need for a non-local formulation in (Bazant, 1991). Generally speaking, a non-local model is based on a theoretical frame requiring the introduction of an internal length parameter. This length can be related to material properties, such as grain size. Bazant and Jirasek (Jirasek, 1998; Bazant and Jirasek, 2002) distinguish microstructure, differential and integral theories. In *microstructure models*, which will be studied in the following subsection, each material point is seen as a deformable particle endowed with degrees of freedom defined at the microscale. The structure of the material is thus enriched. *Differential frameworks* are based on the introduction of the gradients of local or non-local variables in the constitutive equations. Some of them are based on the theory of microstructure. For example, in second grade models, the internal power density is assumed to depend not only on deformations, but also on the gradient of deformations. It is the representation of a particular micromorphic medium of degree one expressed in a first gradient frame, in which macrodeformations and microdeformations are set equal (Germain, 1973b; Vardoulakis and Sulem, 1995; Chambon et al., 2004). In some other differential non-local theories, the spatial gradients are not related to microstructure. Zbib and Aifantis (Al-Holo Al-Radi, 2005) introduced the Laplacian of plastic deformations in a softening behaviour law. In their model, the deviatoric stress depends on the local deviatoric stress and on the first terms of a Taylor's series. In other models, an averaged state variable is replaced by the first terms of the Taylor series of the corresponding local quantity, leading to the introduction of a second-gradient in the constitutive laws (Lasry and Belytschko, 1988; de Borst et al., 1999 ; Kuhl and Ramm, 1999 ; Askes et al., 2000 ; Askes and Sluys, 2002 ; Pamin, 2005 ; Isaksson and Hägglund, 2007). The weighing coefficients of the gradient terms depend on a material length related to the dimension of the zone of influence of local damage. *Integral formulations* (Bazant and Ozbolt, 1990 ; Bazant, 1991 ; de Vree et al., 1995 ; Jirasek, 1998) also involve the spatial averaging of material properties on the neighbourhood of the observed point. As explained in the review paper by Bazant and Jirasek (Bazant and Jirasek, 2002), a local state variable $f(x)$ is replaced by a spatial average $\bar{f}(x)$:

$$\bar{f}(x) = \int_{V_{tot}} \alpha(x,\xi) f(\xi) d\xi \qquad (4)$$

where V_{tot} denotes the volume of the entire system (and not just the REV). The attenuation function $\alpha(x,\xi)$ represents the decreasing distance of influence of a given state variable in space. The use of decreasing weighing functions allows the averaging integral (4) on the whole volume of the system V_{tot}.to be calculated. It is thus not necessary to define representative volume elements explicitly, because homogenisation is already included in the computation process. In order to guarantee the absence of residual stresses after damage, Jirasek (Jirasek, 1998) recommends the introduction of either the non-local deformation, the non-local energy release rate, or the non-local damage deformation into the constitutive relations.

1.1.2. Energetic approaches

Energetic considerations are particularly suited to model dissipative phenomena such as damage and plasticity. Thermodynamic potentials are given specific forms. In many models, the expression for the free energy is chosen depending on the expected behaviour law (Svedberg and Runesson, 1997; Homand-Etienne et al., 1998; Shao and Rudnicki, 2000; Menzel and Steinmann, 2001; Shao, Ata and Ozanam, 2005; Shao, Zhou and Chau, 2005). Formulations starting from the Principle of Virtual Power (Frémond and Nedjar, 1996; Pires-Domingues et al., 1998; Nedjar, 2001; Zhao et al., 2005) can encompass an enrichment of the material's structure, implying the definition of higher-order stresses and specific boundary conditions.

Thermodynamic framework

An overview of continuum thermodynamics is given in (Hansen and Schreyer, 1994) and (Collins and Houlsby, 1997). At a local point $\underline{x}$, the internal energy U of the studied system depends on the entropy $S(\underline{x})$, strain variables $\underline{\underline{E}}(\underline{x})$ and on parameters representing irreversible or dissipative processes $\nu_i(\underline{x})$. The first law of thermodynamics means that the variation of the internal energy is equal to the work of deformation minus the heat provided to the exterior of the system:

$$\dot{U}(S(\underline{x}),\underline{\underline{E}}(\underline{x}),\upsilon_i(\underline{x})) = \underline{\underline{\Sigma}}(\underline{x}):\underline{\underline{\dot{E}(\underline{x})}} - \underline{\nabla}.\underline{q}(\underline{x}) \tag{5}$$

$\underline{\underline{\Sigma}}(\underline{x})$ is the generalized stress tensor, and $\underline{q}(\underline{x})$ is the heat flux vector. Due to the occurrence of irreversible processes, entropy production always exceeds the quantity of heat transmitted to the exterior (Coussy and Dangla, 2002):

$$T(\underline{x})\dot{S}(\underline{x}) \geq -\underline{\nabla}.\underline{q}(\underline{x}) \tag{6}$$

where $T(\underline{x})$ is the temperature of the medium. Combining equations (5) and (6) leads to the Inequality of Clausius-Duhem (ICD):

$$\mathrm{T}(\underline{\mathrm{x}})\dot{\mathrm{S}}(\underline{\mathrm{x}}) \geq \dot{\mathrm{U}}(\mathrm{S}(\underline{\mathrm{x}}),\underline{\underline{\mathrm{E}}}(\underline{\mathrm{x}}),\upsilon_{\mathrm{i}}(\underline{\mathrm{x}})) - \underline{\underline{\Sigma}}(\underline{\mathrm{x}}) : \dot{\underline{\underline{\mathrm{E}}}}(\underline{\mathrm{x}}) \tag{7}$$

For reversible processes, $\dot{\mathrm{v}}_{\mathrm{i}}(\underline{\mathrm{x}}) = 0$ and the ICD is an equality. Due to the independence of the chosen state variables, this statement leads to the equalities:

$$\frac{\partial U(S(\underline{x}),\underline{\underline{\mathrm{E}}}(\underline{x}),\upsilon_i(\underline{x}))}{\partial S(\underline{x})} = T(\underline{x}), \qquad \frac{\partial U(S(\underline{x}),\underline{\underline{\mathrm{E}}}(\underline{x}),\upsilon_i(\underline{x}))}{\partial \underline{\underline{\mathrm{E}}}(\underline{x})} = \underline{\underline{\Sigma}}(\underline{x}) \tag{8}$$

The partial Legendre transform of the internal energy relative to entropy is defined as the Helmholtz free energy $\mathrm{F}(\mathrm{T}(\underline{\mathrm{x}}),\underline{\underline{\mathrm{E}}}(\underline{\mathrm{x}}),\upsilon_{\mathrm{i}}(\underline{\mathrm{x}}))$:

$$\mathrm{U}(\mathrm{S}(\underline{\mathrm{x}})),\underline{\underline{\mathrm{E}}}(\underline{\mathrm{x}}),\upsilon_{\mathrm{i}}(\underline{\mathrm{x}})) - \mathrm{F}(\mathrm{T}(\underline{\mathrm{x}}),\underline{\underline{\mathrm{E}}}(\underline{\mathrm{x}}),\upsilon_{\mathrm{i}}(\underline{\mathrm{x}})) = \mathrm{T}(\underline{\mathrm{x}})\mathrm{S}(\underline{\mathrm{x}}) \tag{9}$$

Behaviour and evolution laws

In phenomenological damage models, behaviour laws are derived from the expression for the free energy. Determining the evolution laws of the irreversible state variables generally requires additional assumptions, such as the existence of a dissipation potential (Svedberg and Runesson, 1997) or the validity of a yield function (Shao and Rudnicki, 2000; Shao, Ata and Ozanam, 2005). If flow rules are non-associated, a potential has to be introduced in addition to the yield function (Vardoulakis, 2000; Tamagnini et al., 2001). Plastic and damage multipliers are computed by means of the consistency equation dealing with the corresponding yield function. The complementary conditions of Kuhn-Tucker must also be satisfied.

Applying the maximal dissipation principle (damage models: Pires-Domingues et al., 1998; Menzel and Steinmann, 2001; plastic models: Chambon et al., 2004) avoids the need to assume the form of the evolution laws. At a stable equilibrium state, “the entropy production rate is minimized while the entropy reaches a maximum for a given total energy” (Hansen and Schreyer, 1994). Assuming that the thermodynamic state is characterized by the strain tensor $\underline{\underline{\varepsilon}}$ and a damage variable $\underline{\underline{\Omega}}$, the reduced dissipation inequality can then be written as:

$$\underline{\underline{Y}} : \dot{\underline{\underline{\Omega}}} \geq 0, \qquad \underline{\underline{Y}} = \frac{\partial F(\underline{\underline{\varepsilon}}, \underline{\underline{\Omega}})}{\partial \underline{\underline{\Omega}}}, \qquad \underline{\underline{\sigma}} = \frac{\partial F(\underline{\underline{\varepsilon}}, \underline{\underline{\Omega}})}{\partial \underline{\underline{\varepsilon}}} \tag{10}$$

in which $F(\underline{\underline{\varepsilon}}, \underline{\underline{\Omega}})$ is the Helmholtz free energy for isothermal conditions. The principle of maximum dissipation, or principle of maximum entropy, thus consists in maximizing $\underline{\underline{Y}} : \dot{\underline{\underline{\Omega}}}$. Assuming that the damage and hardening variables obey a criterion given by the yield function $F_d(\underline{\underline{\sigma}}, \underline{\underline{Y}})$, the maximization problem can be solved as a constrained minimization problem, where the function to minimize is:

$$L(\underline{\underline{\sigma}}, \underline{\underline{Y}}) = -\underline{\underline{Y}} : \dot{\underline{\underline{\Omega}}} + \dot{\lambda}_d F_d(\underline{\underline{\sigma}}, \underline{\underline{Y}}) \tag{11}$$

with the optimality conditions of Kuhn-Tucker:

$$\dot{\lambda}_d \geq 0, \qquad F_d(\underline{\underline{\sigma}}, \underline{\underline{Y}}) \leq 0, \qquad \dot{\lambda}_d F_d(\underline{\underline{\sigma}}, \underline{\underline{Y}}) = 0 \tag{12}$$

The Lagrangian coefficient $\dot{\lambda}_d$ can be regarded as the damage multiplier. Among all the admissible states, the equilibrium state minimizes $L(\underline{\underline{\sigma}}, \underline{\underline{Y}})$. The first-order derivatives of $L(\underline{\underline{\sigma}}, \underline{\underline{Y}})$ relative to $\underline{\underline{\sigma}}$ and $\underline{\underline{Y}}$ are consequently equal to zero, which results in the evolution law of damage.

The Lagrangian resolution of the problem of maximum dissipation thus leads to associative flow rules (Hansen and Schreyer, 1994). It is still possible to add conditions in order to model a non-associated behaviour. In the gradient plasticity model of Chambon (Tamagnini et al., 2001; Chambon et al., 2004), the flow rules and the hardening law obtained by the Lagrangian resolution are changed by replacing the yield function F_p involved in the formulae by a plastic potential G_p and a hardening function R_p. F_p, G_p and R_p are assumed to depend on the same state variables. The main advantage of the application of the maximum dissipation principle is that the ICD is always satisfied, so that the resulting model is automatically thermodynamically consistent.

Structure enrichment

Some conditions on the form of the internal power density may be set before assuming the expression of the free energy. By doing so, it is possible to change the global form of the principle of virtual power, which influences the formulation of the balance equations. Moreover, the model of the material structure may be affected by the introduction of gradient variables in the expression of the internal power. The theory of microstructure introduced by Mindlin (Mindlin, 1964) is based on three basic assumptions, viz:

1. the internal power of the system depends on displacements and on the gradient of displacements;

2. the material can be represented with a microstructure, which implies that every material point is seen as a deformable particle, endowed with microscopic degrees of freedom. As a result, the internal power density of the body depends on macroscopic displacements U_i, on the macro-gradient of macrodisplacements (or macrodeformations) $\partial_j U_i$, on microdisplacements U_i' and on the micro-gradient of microdisplacements (or microdeformations) $\partial_j' U_i'$;

3. microdisplacements U_i' can be expanded in a Taylor's series, relative to the relative position x_i'. Mindlin retained only the first two terms of this series:

$$U_i' = U_i + \chi_{ij} x_j' \tag{13}$$

As there is a linear dependence between microscopic displacements and the relative position, the medium is said to be a micromorphic continuum of degree one. As macrodeformations do not vary with the relative position, the form of the internal power density assumed by Mindlin depends only on U_i, $\partial_i U_i$, $\eta_{ij} = \partial_i U_i - \partial_j 'U_i'$ and $\chi_{ij,k} = \partial_k \chi_{ij}$, $\chi_{ij} = \partial_j 'U_i'$. As recalled in (Germain, 1973a), the application of the Virtual Power Principle requires the assumption that the virtual power of internal stresses developed by the system in a rigid body motion is equal to zero, which infers that the U_i-related term of the internal power disappears. The internal power density thus takes the following form:

$$p_{int} = \sigma_{ij} \partial_j U_i + s_{ij} \eta_{ij} + \nu_{ijk} \chi_{ij,k} \tag{14}$$

Second gradient models are defined as micromorphic Mindlin continua of degree one, with the additional assumption that microdeformations $\partial_j 'U_i'$ are equal to macrodeformations $\partial_j U_i$. The internal power density (14) thus depends only on $\partial_j U_i$ and $\chi_{ij,k} = \partial_k \chi_{ij} = \partial_k \partial_j 'U_i' = \partial_k \partial_j U_i$. In other words, the formulation depends only on macrodeformations and on the gradient of macrodeformations. Vardoulakis and Sulem (Vardoulakis and Sulem, 1995; Vardoulakis, 2000) used the microstructure frame to develop a behaviour model for granular dilatant rocks. Chambon and his co-workers (Matsushima et al., 2000; Chambon et al., 2001; Tamagnini et al., 2001 ; Chambon et al., 2004) studied multimechanism plasticity for frictional cohesive materials through a second gradient model. Frémond (Frémond and Nedjar, 1996) also enriched the structure of the medium by introducing the gradient of damage in the expression of the internal power of the system. The gradient of damage plays the same role as the gradient of macrodeformations and requires the definition of higher-order terms in the application of the Virtual Power Principle. Other researchers followed the same reasoning, e.g. Pires-Domingues (Pires-Domingues et al., 1998), who studied non-linear elastic brittle materials, and Nedjar (Nedjar, 2001), who coupled the damage model of Frémond to an elastoplastic theory. Zhao and his co-workers (Zhao et al., 2005) based their model of coupled plasticity and damage on second gradient theory, including the gradient of deformations in the internal power and the gradient of the hardening variable in the expression of the free energy. In all cases, the presence of gradients in the surface terms of the expression of power imposes corresponding higher-order boundary conditions. For this purpose, spatial derivation of virtual displacements is usually split into a normal operator and a tangential operator.

1.2. **Theoretical relevance of the various models**

1.2.1. Physical meaning and mechanical representation of damage

Physical meaning of the damage variable

Based on the definition of one or two thermodynamic potentials, phenomenological models avoid the manipulation of huge quantities of microscopic parameters, which accelerates numerical computations. However, they are more focused on the mechanical effects of damage than on the representation of cracking. The whole theoretical frame is devoted to the derivation of behaviour laws. Damage evolution is often determined by an associated flow rule, which is not an exclusive tool of thermodynamic modelling. On the contrary, micromechanical concepts are aimed at relating the damage variable to a physical description of material degradation. Bazant and Ozbolt proposed a micro-planar representation of damage (Bazant and Ozbolt, 1990). In the non-local formulation presented by Lacy (Lacy et al., 1999), microscopic cracking data are homogenised. In mixed models, fractures are often gathered in families of roughly parallel cracks. After one or several homogenisation processes, the principal directions of the damage tensor are known, which is generally sufficient to represent anisotropy (Homand-Etienne et al., 1998; Shao and Rudnicki, 2000; Shao, Ata and Ozanam, 2005; Swoboda and Yang, 1999b). Even scalar micromechanical damage variables are given a physical meaning: damage is defined as the ratio of effective material volume relatively to the volume the initial intact sample (de Borst et al., 1999), or as an energy release rate (Bazant, 1991). The more “physical” phenomenological models resort to gradient-enhancement (Frémond and Nedjar, 1996; Pires-Domingues et al., 1998; Nedjar, 2001), which introduces a form of fracture anisotropy. This regularization technique is also used in micro-mechanical theoretical frames (de Borst et al, 1999; Askes et al., 2000; Askes and Sluys, 2002; Pamin, 2005).

Loss of rigidity and anisotropy of damage

Generally speaking, micromechanical models using scalar variables account for the effective surface or volume of material, and encompass a yield function representing strain softening (de Borst et al., 1999; Askes et al., 2000; Askes and Sluys, 2002; Pamin, 2005). In theories resorting to a tensorial damage variable, damage evolution is generally related to a fracturing criterion, as in the micromechanical part of the formulation of some mixed models (Homand-Etienne, 1998; Shao and Rudnicki, 2000; Shao, Zhou and Chau, 2005; Swoboda and Yang, 1999b).

In phenomenological models, the relation between damage increase and mechanical degradation is abstract, because the behaviour laws are deduced from postulated thermodynamic potentials. Fracturing anisotropy is directly introduced in the stiffness matrix. The computation is easier than in micromechanical damage models, in which the damage parameters have to be explicit.

Reversibility of damage

In all the preceding theories, damage is associated to irreversibility. However, some degraded mechanical properties may be recovered. In concrete, damage generated by tensile stress may be compensated by compression. That is why the approach of Mazars (Mazars, 1984), which consists of splitting a scalar damage variable into a reversible (tensile) and an irreversible (compressive) part, has been used in both micro-mechanical (Bourgeois et al., 2002) and phenomenological (Frémond and Nedjar, 1996) models. The phenomenological theory developed by Frémond and Nedjar allows the representation of another form of reversibility, because the work-conjugate variable associated with damage is split into a reversible and an irreversible part.

Due to the induced computational complexity, tensorial micro-mechanical formulations are not suited to the representation of crack-closure. Halm and Dragon proposed a phenomenological model resorting to two second-order damage tensors instead of one, in order to represent unilateral effects in cohesive materials (Halm and Dragon, 1998). Bargellini advocates the use of a single fourth-order tensorial damage. Damage directions are fixed, and associated to scalar damage densities. (Bargellini et al., 2006).

Alternatively, the mixed model of Hou (Hou, 2003) is based on an additive breakdown of deformations. One component represents deformations generated by damage growth, and another models healing effects. But the formulation proposed by Hou requires a lot of material parameters.

1.2.2. Thermodynamic consistency of the theory

Phenomenological models built on the maximization of dissipation are automatically thermodynamically consistent. In other energetic approaches, the Inequality of

Clausius-Duhem has to be satisfied as an additional condition (Frémond and Nedjar, 1996; Svedberg and Runesson, 1997; Shao and Rudnicki, 2000; Nedjar, 2001), since the dissipation inequality is not part of the formulation. In mere micromechanical models (de Borst et al., 1999; Pamin, 2005; Askes et al., 2000; Askes and Sluys, 2002; Bazant, 1991; de Vree et al., 1995; Jirasek, 1998; Bazant and Ozbolt, 1990), thermodynamic considerations are generally avoided. However, Kuhn-Tucker complementary conditions have to be satisfied in all types of models. These inequalities are basically related to dissipation represented by the evolution process of yield surfaces (Hansen and Schreyer, 1994).

1.2.3. Mathematical regularization of localization

Regularization techniques in mechanical damage models

As stressed in (Lasry and Belyschko, 1988; Lorentz and Andrieux, 1999; Askes et al., 2000; Askes and Sluys, 2002; Kuhl and Ramm, 1999), strain softening cannot be represented by a classical model of Continuum Damage Mechanics. For very high stresses, deformations tend to concentrate in a narrow zone, the width of which depends on the mesh size. Moreover, the dissipated energy goes to zero, which is, physically, nonsense. From a mathematical point of view, the problem becomes ill-posed. In quasi-static problems, the ellipticity of the governing equations is lost, while dynamical hyperbolic equations become elliptic (Lasry and Belytschko, 1988). In the theory of plasticity, the localization of deformations into shear bands has been studied extensively in (Belytschko and Kulkarni, 1990; Vardoulakis and Sulem, 1995; Vardoulakis, 2000; Chambon et al., 2001; Tamagnini et al., 2001; Chambon et al., 2004). As explained in (Chambon et al., 2004), the modelling of softening behaviours requires the introduction of an internal length scale characterizing the medium. Lasry and Belytschko (Lasry and Belytschko, 1988) describe four ways to limit localization phenomena: i) modelling of microstructure; ii) spatial averaging of state variables (integral non-local formulation); iii) introduction of the spatial gradients of the state variables (differential non-local formulation); and iv) regularization by a material rate dependency, for example, by means of a viscoplastic model.

Micromechanical models usually resort to spatial averaging (Lacy et al., 1999; Bazant, 1991; de Vree et al., 1995; Jirasek, 1998; Bazant and Ozbolt, 1990) or gradient-enhancement (de Borst et al., 1999; Pamin, 2005; Askes et al., 2000; Askes and Sluys, 2002). Microstructure approaches are more frequent in plasticity models (Vardoulakis and Sulem, 1995; Chambon et al., 2004), but can be found in phenomenological damage theories (Frémond and Nedjar, 1996; Pires-Domingues, 1998; Nedjar, 2001; Zhao et al., 2005). Integral non-local formulation often stems from a homogenisation or averaging process, which is not really suited for phenomenological models, which are not based on a fracturing-related definition of damage. On the contrary, space gradients of state variables are often included in energetic models of damage (Kuhl and Ramm, 1999; Svedberg and Runesson, 1997; Isaksson and Hägglund, 2007).

The internal material length parameter appears during the mathematical development of the non-local formulation. In gradient-enhanced models, the material length is generally related to a Taylor's series expansion (Askes et al., 2000). Spatial integrations involve either the characteristic dimension of the Representative Elementary Volume (Bazant, 1991; de Vree et al., 1995; Lacy et al., 1999) or a distance of influence, if attenuation functions are defined (Bazant and Ozbolt, 1990; Jirasek, 1998). The characteristic size of the REV must be chosen to allow mechanical homogenisation. In the model of Homand-Etienne and her co-workers (Homand-Etienne et al., 1998), the REV length is used to define a criterion ensuring that the sample can always be seen as a continuum.

Regularization term in the expression of the free energy

Similarities may be found in the expressions of the free energy used in regularized models. Relating the damage variable $\underline{\underline{\Omega}}$ to an equivalent mechanical state defined at the scale of a REV, the expression of Helmholtz free energy used in the mixed model of Swoboda and Yang (Swoboda and Yang, 1999a) becomes:

$$F = \frac{1}{2}\underline{\underline{\varepsilon}} : \underline{\underline{\underline{\underline{D}}}}(\underline{\underline{\Omega}}) : \underline{\underline{\varepsilon}} - g\underline{\underline{\Omega}} : \underline{\underline{\varepsilon}} \tag{15}$$

$g\underline{\underline{\Omega}} : \underline{\underline{\varepsilon}}$ results from a homogenisation process, and is thus related to the regularization of strain (or crack) localization. A very similar term appears in the expression for the free energy defined in the mixed models of Shao (Homand-Etienne, 1998; Shao, Ata and Ozanam, 2005).

Considering that "damage, or the generation and propagation of micro-defects in the material, causes micro-cracks and micro-surfaces to grow" and that "the increase in the size of material surfaces corresponds to an increase in the material surface energy" (Hansen and Schreyer, 1994), a surface energy term $\mathrm{G_D}(\Omega)$ may be introduced in the general expression of the free energy of a damaged material:

$$\mathrm{G_D}(\underline{\underline{\Omega}}) = \gamma_{\mathrm{D}}\underline{\underline{\Omega}} : \underline{\underline{\Omega}} \tag{16}$$

in which γ_{D} is a material constant. In micromechanical models, damage is related to the size and opening of cracks, and has generally the dimension of a deformation. Therefore, the expression of $\mathrm{G_D}(\Omega)$ proposed in equation (16) is very close to the regularization term $g\underline{\underline{\Omega}} : \underline{\underline{\varepsilon}}$ appearing in mixed formulations.

In the phenomenological microstructure model of Frémond and Nedjar (Frémond and Nedjar, 1996), the free energy encompasses a term which is very similar to the surface energy defined by Hansen and Schreyer in equation (16):

$$\frac{k}{2}\left(\underline{\nabla}\omega\right)^2 \equiv \gamma_D \underline{\underline{\Omega}} : \underline{\underline{\Omega}} \qquad (17)$$

k is defined as a measure of the influence of the damage ω developed at a material point on the damage characterizing the neighbourhood of this material point. k is given the dimension of a force, i.e. a stress multiplied by the square of a length. The length implicitly defined through the k parameter quantifies the distance of influence of local damage. It can be compared to the attenuation functions used in the non-local models based on an integral formulation (Bazant and Ozbolt, 1990; Jirasek, 1998). $\frac{k}{2}\left(\underline{\nabla}\omega\right)^2$ may thus be considered as a regularization term.

Similarly, in the phenomenological gradient-enhanced model of Kuhl and Ramm (Kuhl and Ramm, 1999), a regularization-related elastic potential appears in the expression of the free energy:

$$F^{\Omega} = \frac{c}{2}\left(\underline{\nabla} \cdot \bar{\underline{\underline{\varepsilon}}}\right)^2 \qquad (18)$$

$\bar{\underline{\underline{\varepsilon}}}$ is a non-local state variable depending on damaged stress. c is a damage parameter with the dimension of the square of a characteristic length.

Lastly, in the second-gradient plasticity model designed for cohesive frictional materials by Chambon (Chambon et al., 2004), the assumed split expression of the free energy involves a microstructural term, the form of which is very similar to the regularization term already used in the microstructure model of Frémond and Nedjar:

$$W_2\left(\underline{\underline{\varepsilon}}, \underline{\underline{\underline{\nabla}}}\left(\underline{\underline{\varepsilon}}\right), \underline{\underline{\underline{\nabla}}}\left(\underline{\underline{\varepsilon}}^p\right)\right) = \frac{\mu \cdot l^2}{2}\left(\underline{\underline{\underline{\nabla}}}\left(\underline{\underline{\varepsilon}}^p\right)\right)^2 \equiv \frac{k}{2}\left(\underline{\nabla}\omega\right) \qquad (19)$$

μ is the classical shear modulus, with the dimension of stress, and l is a characteristic length related to the plastic behaviour law. $\underline{\underline{\underline{\nabla}}}\left(\underline{\underline{\varepsilon}}^p\right)$ is the gradient of a microstructure variable related to a dissipation process, exactly like $\underline{\nabla}\omega$. $\mu \cdot l^2$ thus plays the same role as k, and quantifies the influence of the plastic behaviour of a material point on its neighbourhood.

In summary, the free energy expressed in regularized models of plasticity and damage always encompass a regularization term involving a material length related to homogenisation processes or to a distance of influence of local mechanical behaviour:

$$g\underline{\underline{\Omega}} : \underline{\underline{\varepsilon}},\ \gamma_D \underline{\underline{\Omega}} : \underline{\underline{\Omega}},\ \frac{k}{2}\left(\underline{\nabla}\omega\right)^2,\ \frac{c}{2}\left(\underline{\nabla}\cdot\bar{\underline{\underline{\varepsilon}}}\right)^2,\ \frac{\mu\cdot l^2}{2}\left(\underline{\underline{\nabla}}\left(\underline{\underline{\varepsilon}}^p\right)\right)^2 \tag{20}$$

1.3. Extending damage models to unsaturated materials

1.3.1. Solutions proposed in the literature

Biot's effective stress concept

For dry materials, the damaged stress-strain relation results from the expression of the degraded stiffness. The latter is provided by application of the Principle of Equivalent Elastic Energy (PEEE) in micro-mechanical models, and by the determination of conjugation relations in phenomenological models. For saturated media, a clear separation of mechanical and hydraulic effects may be made through the definition of Biot's effective stress. The damaged stress-strain relation that is used for total stress in a dry material can be applied to effective stress. For unsaturated media, this reasoning requires the definition of an equivalent pore pressure, in order to represent the influence of fluids on mechanical stress (Shao, Ata and Ozanam, 2005):

$$\underline{\underline{\sigma}} = \frac{\partial F(\underline{\underline{\varepsilon}},\underline{\underline{\Omega}})}{\partial \underline{\underline{\varepsilon}}} - b\left[S_w p_w + (1-S_w)p_g\right]\ \underline{\underline{Id}} \tag{21}$$

b is Biot's hydro-mechanical coupling coefficient, S_w is the saturation degree of the liquid fluid phase, p_w is the liquid pore pressure and p_g is the gas pore pressure. In the elastoplastic damage models of Shao, Bourgeois and Duveau (Bourgeois et al., 2002; Shao et al., 2006; Jia et al., 2007), a Biot's type plastic stress is defined. The Biot's type coefficients implied in the expressions of total and plastic stresses are not necessarily equal. Total stress is changed into effective stress by introduction of a damaged rigidity. Both hydraulic and damage effects control the mechanical state variable. But there is no coupling between the two. The same type of formulation is adopted in (Shao, Ata, Ozanam, 2005), though plasticity is not modelled. Jia (Jia et al., 2007) uses two scalar damage variables to model brittle materials, in which damage occurs before plasticity. One affects plastic stresses and the other is directly involved in the constitutive relation. Mechanical dissipation processes affect the intrinsic permeability, which is assumed to depend on the plastic hardening parameter. In other words, stresses have to exceed both damage and plastic thresholds to influence fluid transfers.

The preceding constitutive relations, based on Biot's representation of stress, uncouple poromechanical and damage effects. Capillarity effects on deformation are neglected. Damage growth is still synonymous with increase in fracturing. Defect initiation or crack aperture generates an increase in pore size at the scale of the global network of the equivalent medium. Larger pores induce smaller capillarity effects, and consequently, a lower rigidity (Gatmiri et al., 1997 ; Gatmiri 2002). Conversely, suction is work-conjugated to the partial porosity of the liquid phase (Houlsby, 1997), which introduces hydraulic effects in the mechanical behaviour. This is why a formulation based on net stress and suction might be more satisfying from a conceptual point of view.

Independent stress state variables

To the authors' knowledge, very few damage models of unsaturated media are formulated in net stress and suction. Lu and his co-workers (Lu et al., 2006) proposed to split total stresses between a relatively damaged part and a relatively intact part, with the contributions of each weighed by a scalar damage variable depending on deviatoric strains and suction. Assuming that strain and suction change consistently during loading in the relatively intact and relatively damaged regions, it becomes possible to relate total stresses to the equivalent deformation tensor and to the equivalent suction. Contrary to a mere effective stress concept, the damaged regions of the material are still submitted to stresses, even if these "damaged stresses" do not follow the same stress/strain relations as the "undamaged stresses". The model of Lu and his co-workers (Lu et al., 2006) can easily be extended to anisotropic damage. However, the approach is merely micromechanical and thermodynamic requirements are not considered.

1.3.2. A new hydro-mechanical damage model for unsaturated porous media

In order to take advantage of both micro-mechanical and phenomenological theoretical frames, a new damage model has been formulated in terms of net stress $\underline{\underline{\sigma}}'' = \underline{\underline{\sigma}} - p_g \underline{\underline{Id}}$ and suction $s = p_g - p_w$. The damage variable $\underline{\underline{\Omega}}$ is a second-order tensor, expressed in terms of its spectral decomposition (See equation 3). It is assumed that the total deformation tensor is composed of reversible and irreversible stress-conjugated strains:

$$d\underline{\underline{\varepsilon}} = d\underline{\underline{\varepsilon}}_M^{rev}\left(\underline{\underline{\Omega}}\right) + d\underline{\underline{\varepsilon}}_S^{rev}\left(\underline{\underline{\Omega}}\right) + d\underline{\underline{\varepsilon}}_M{}^{irr} + d\underline{\underline{\varepsilon}}_S{}^{irr} \tag{22}$$

with the following assumptions:

$$\begin{cases} d\underline{\underline{\varepsilon}}_M^{rev}\left(\underline{\underline{\Omega}}\right) = \underline{\underline{\underline{\underline{D_e}}}}^{-1}\left(\underline{\underline{\Omega}}\right) \quad : d\underline{\underline{\sigma}}'' \\ d\underline{\underline{\varepsilon}}_S^{rev}\left(\underline{\underline{\Omega}}\right) = \dfrac{1}{3} d\varepsilon_{Sv}^{rev}\left(\underline{\underline{\Omega}}\right)\underline{\underline{Id}} = \dfrac{1}{3\beta_s\left(\underline{\underline{\Omega}}\right)}\underline{\underline{Id}} \quad ds \end{cases} \tag{23}$$

Using the operator of Cordebois and Sidoroff (Cordebois and Sidoroff, 1982), damaged net stress is defined, and the concept is extended to suction:

$$Tr\left(\hat{s} \quad \underline{\underline{Id}}\right) = Tr\left(\underline{\underline{\underline{\underline{M}}}}\left(\underline{\underline{\Omega}}\right) : \underline{\underline{Id}} \quad s\right) = s \quad Tr\left(\left(\underline{\underline{Id}} - \underline{\underline{\Omega}}\right)^{-1/2} . \ \underline{\underline{Id}} \ . \ \left(\underline{\underline{Id}} - \underline{\underline{\Omega}}\right)^{-1/2}\right) \tag{24}$$

The PEEE is applied for both stress state variables, which provides the damaged rigidities. The increment of damage is computed by an associated flow rule. Incremental irreversible strains are expressed as functions of incremental damage by means of conjugation relations, which are derived from the following expression for the free energy:

$$F\left(\underline{\underline{\varepsilon}}_M, \varepsilon_{S_v}, \underline{\underline{\Omega}}\right) = \frac{1}{2}\underline{\underline{\varepsilon}}_M : \underline{\underline{\underline{\underline{D_e}}}}\left(\underline{\underline{\Omega}}\right) : \underline{\underline{\varepsilon}}_M + \frac{1}{2}\varepsilon_{S_v}\beta_s\left(\underline{\underline{\Omega}}\right)\varepsilon_{S_v} - g_M \underline{\underline{\Omega}} : \underline{\underline{\varepsilon}}_M - \frac{g_S}{3} Tr\left(\underline{\underline{\Omega}}\right)\varepsilon_{S_v} \tag{25}$$

Definition (25) is an extension of the model of Dragon and Halm (Dragon and Halm, 1996) to unsaturated materials. The free energy encompasses a regularization term for each stress-state variable.

2. Hydraulic properties of a fractured porous medium

Transfer has been extensively studied through models of porous networks. The main differences between the theories lie in the number of represented continua, and in the modelling of fluid exchanges between continua. In the following, attention will be focused on liquid transfers in an unsaturated fractured material. After setting the equilibrium equations, the modelling of retention properties will be tackled. The modelling of fluid transfers in a damaged material considered as a single continuum is still an open subject, whose coupling problems will be discussed in the last part of this section.

2.1. **Liquid transfer equations**

2.1.1. Multimodal models

In multimodal models (Durner, 1994), the porous structure of the medium is assumed to consist of communicating identified networks. The pores of the intact matrix and the cracks generated by damage growth form a unique porous network. A unique pressure head h represents the hydraulic state of the various pore systems. Consequently, liquid transfer is formulated by means of a single Richard's equation:

$$\frac{\partial \theta_w(h)}{\partial h} \cdot \frac{\partial h}{\partial t} = \nabla \cdot \left(\underline{\underline{K_w}}(h) \cdot \underline{\nabla}(h+z) \right) \tag{26}$$

$\theta_w(h)$ and $\underline{\underline{K_w}}(h)$ are the water content and water permeability of the equivalent fractured porous medium respectively. The form of balance equation (26) is the same as the one that would have been obtained for a single fracture network (Zimmerman and Bodvarsson, 1996; Liu and Bodvarsson, 2001) or for a single matrix porous system (Van Genuchten, 1980; Luckner et al., 1989).

Pruess and his co-workers (Pruess et al., 1990) defined the equivalent continuum as the medium characterized by the same fluxes, and the same temperature, pressure and saturation distributions as the discretized system made of a porous matrix and a fracture network, under given initial and boundary conditions. In the REV, the retention properties of matrix and fractures considered as separate networks are homogenised in order to compute the retention property $\theta_w(h)$ of the equivalent porous continuum. This is discussed in the following paragraph. The permeability matrix $\underline{\underline{K_w}}(h)$ is then deduced from a postulated relation between equivalent water content and equivalent permeability $\underline{\underline{K_w}}(\theta_w(h))$.

2.1.2. Multi-continua models

In multi-continua models, several porous networks drive the flow and behave as separate entities. A fractured porous material is frequently modelled as a dual continuum. The constitutive pores of the intact matrix form the first porous network, and the cracks, assumed to form a connected network, constitute the second porous system. If fluid exchanges occur at the interfaces between matrix and fractures, the Richard's equations governing the mass balance of fluid are coupled, and the pressure head has to be defined in each porous continuum (Othmer et al., 1991; Gwo et al., 1995; Gerke and Van Genuchten, 1993; Vogel et al., 2000; Zimmerman et al., 1996):

$$\begin{cases} (1-w_f)\dfrac{\partial\theta_{w,m}(h_m)}{\partial h_m}\cdot\dfrac{\partial h_m}{\partial t} = (1-w_f)\underline{\nabla}\cdot\left(\underline{\underline{K_{w,m}}}(h_m)\cdot\underline{\nabla}\left(h_m+z\right)\right)-\Gamma_w \\ w_f\dfrac{\partial\theta_{w,f}(h_f)}{\partial h_f}\cdot\dfrac{\partial h_f}{\partial t} = w_f\underline{\nabla}\cdot\left(\underline{\underline{K_{w,f}}}(h_f)\cdot\underline{\nabla}\left(h_f+z\right)\right)+\Gamma_w \end{cases} \tag{27}$$

m and f subscripts refer to the intact matrix porous network and to the fracture network respectively. w_f is the ratio of the volume occupied by fractures relative to the total volume of the representative element (Pruess et al., 1990; Gerke and Van Genuchten, 1993). The transfer parameter Γ_w is often modelled as a first-order interaction coefficient (Gerke and Van Genuchten, 1993; Othmer et al., 1991; Vogel et al., 2000).

A penetration time can be defined (Pruess et al., 1990; Zimmerman et al., 1996), to indicate when suction equilibrium occurs, and thus, when it is possible to consider one single pressure head variable for the whole REV. Alternatively, some authors (Köhne et al., 2002; Cey et al., 2006) consider that thermodynamic equilibrium is reached immediately at interfaces, and directly use decoupled Richard's equations, involving a single pressure head for the whole system.

2.2. **Hydraulic retention curves**

2.2.1. Heterogeneous and multimodal porosity models

In multimodal models, a unique pore pressure variable represents the hydraulic state of the various pore systems. But each porous network is characterized by its pore size and its retention properties $\theta_{wi}(h)$. The relative organization of porous heterogeneity makes it impossible to express the global hydraulic properties of the REV by a mere Van Genuchten-Mualem model, based on a Bell-shaped distribution of pore sizes among a single porous network (Van Genuchten, 1980). In the model of Durner (Durner, 1994), the adimensional water content of the global medium $\Theta(h)$ is deduced from a weighed integration of the various individual retention curves. The weighting coefficients are determined by fitting parameters of the model to experimental data, through the minimization of a target function. The relative permeability of the medium is then computed by a Mualem integral.

2.2.2. Multi-continua systems

In multi-continua models, retention properties have to be defined for each porous system. One possible approach is to apply Van Genuchten formulas (Van Genuchten, 1980) to each continuum, with distinct values for the $\theta_{ws,i}$, $\theta_{wr,i}$, α_i, n_i, m_i parameters. However, some authors consider that flow may be considered at the REV scale. All of the “multi-porosity” models (Zimmerman et al., 1996; Liu and Bodvarsson, 2001) adopt this approach. For example, when one of the continua is known to be much more conductive than the other, the highest permeability is generally used to characterize the equivalent medium. On the other hand, when all continua contribute to the flow, water permeability has to be defined for each porous network. Such situations require a “multi-permeability” model (Othmer et al., 1991; Gwo et al., 1995; Pruess et al., 1990; Köhne et al., 2002; Gerke and Van Genuchten, 1993; Vogel et al., 2000; Cey et al., 2006; Liu et al., 1998; Liu et al., 2004).

The hydraulic properties of the equivalent medium (at the REV scale) can be deduced from the hydraulic parameters of the individual networks by a homogenisation technique. The main difficulty lies in the pore pressure dependency of the equivalent hydraulic properties. If homogenisation is done at thermodynamic equilibrium (Pruess et al., 1990; Gerke and Van Genuchten, 1993; Köhne et al., 2002), pressure heads are the same in all continua.

$$\begin{cases} \theta_{w,REV}(h) = w_f \theta_{w,f}(h) + (1-w_f)\theta_{w,m}(h) \\ k_{R,REV}(h) = w_f k_{R,f}(h) + (1-w_f) k_{R,m}(h) \end{cases} \tag{28}$$

If thermodynamic equilibrium is not assumed (Othmer et al., 1991; Vogel et al., 2000), the equations of hydraulic state surfaces depend on the balance equations, which are coupled by a transfer parameter (See equations 27):

$$\begin{cases} \theta_{w,REV}(h) = w_f \theta_{w,f}(h_f) + (1-w_f)\theta_{w,m}(h_m) \\ k_{R,REV}(h) = w_f k_{R,f}(h_f) + (1-w_f) k_{R,m}(h_m) \\ (1-w_f)\dfrac{\partial \theta_{w,m}(h_m)}{\partial h_m} \cdot \dfrac{\partial h_m}{\partial t} = (1-w_f)\underline{\nabla} \cdot \left(\underline{\underline{K_{w,m}}}(h_m) \cdot \underline{\nabla}\left(h_m + z\right)\right) - \Gamma_w \\ \quad w_f \dfrac{\partial \theta_{w,f}(h_f)}{\partial h_f} \cdot \dfrac{\partial h_f}{\partial t} = w_f \underline{\nabla} \cdot \left(\underline{\underline{K_{w,f}}}(h_f) \cdot \underline{\nabla}\left(h_f + z\right)\right) + \Gamma_w \end{cases} \qquad (29)$$

2.3. **Introducing damage in hydraulic properties**

In network theories, permeability is generally deduced from the water content by an integration technique. Such approaches are merely hydraulic, and the retention functions depend on the pressure head and on fixed hydraulic parameters only. Alternatively, hydro-mechanical frameworks may encompass fully coupled equations for the state surfaces of the void ratio and the saturation degree, as in the model proposed by Gatmiri (Gatmiri et al., 1997). But poromechanics do not represent fracturing. The following paragraph describes a hydraulic model which encompasses all of the couplings involved in a fractured porous medium.

2.3.1. Existing formulas for the damaged intrinsic permeability

The research team of Tang, Tham and Yang (Tang et al., 2002; Yang et al., 2007) have proposed a permeability model for saturated fractured rocks. Effective stresses are defined according to Biot's theory. The intrinsic permeability is defined to be dependent on three damage-related parameters. One of these parameters is Biot's hydro-mechanical coupling coefficient, which is related to a scalar damage variable through the expression of the damaged Young modulus. In the model of Shao (Shao, Zhou, Chau, 2005), micro-crack flow velocities are computed by the cubic law. The expression of the intrinsic permeability is deduced from a summation of flow velocities at the scale of the REV.

2.3.2. A new representation of hydro-mechanical couplings for diffusive liquid flows

Hydro-mechanical couplings cannot be fully represented simply by introduction of damage parameters in the expression for the intrinsic permeability. If Darcy's law is used to model liquid transfers, the introduction of a damage-dependent expression for the relative permeability could fill the gap. Since the relative permeability results from the integration of retention properties, the saturation degree or the water content has then to be related to damage in addition to the pressure head. To the authors' knowledge, such state surfaces do not exist in the literature. Moreover, in models based on a formulation in net stress and suction, suction is seen as a stress variable, that can thus interact with damage.

One possible approach is to adapt a coupled hydro-mechanical state surface to damaged conditions, for example by replacing real state variables by their damaged counterparts. Using the equation of state surface of saturation degree proposed by Gatmiri (Gatmiri et al., 1997 ; Gatmiri, 2002), applying a damage-stress operator to net stress and suction leads to:

$$S_w\left(\underline{\underline{\hat{\sigma}''}},\hat{s}\right) = 1 - \left[a_s + b_s \hat{\sigma}''_m\right] \cdot \left[1 - \exp\left(c_s \hat{s}\right)\right] \tag{30}$$

in which $\hat{\sigma}''_m$ is the mean damaged net stress. Assuming that the liquid flow is diffusive, the combined use of the intrinsic permeability developed by Shao (paragraph 2.3.1), with the relative permeability integrated from equation (30), provides a fully coupled liquid transfer model for a damaged unsaturated porous medium.

Conclusion

Micromechanical damage models are based on effective mechanical concepts, crack characteristics and fracturing criteria. Phenomenological approaches start from a postulate for the expression of the free energy of the medium. Some energetic formulations integrate the principle of maximal dissipation in the computation of the behaviour, flow and hardening/softening laws. The advantage of this process is that the damage model is automatically thermodynamically consistent.

As damage often induces strain softening, crack deformations tend to concentrate in a narrow zone. Localization limiters have to be included to the damage theory in order to avoid an ill-posed problem. A non-local formulation is generally adopted, in order to involve an internal length parameter in the equations of the model. Micromechanical formulations often involve homogenisation or space averaging of selected state variables. Alternatively, the spatial gradient of a given state variable may be introduced into the constitutive relations. This technique has been employed in both phenomenological and micromechanical damage models. Last but not least, some authors have resorted to microstructure enrichment to regularize their energetic damage model. In all regularized formulations, the expression of the free energy encompasses a localization term.

The main issue remains the introduction of hydro-mechanical couplings in damage theories and the prediction of transfer parameters in a damaged material. Porous network models are adapted to determine hydraulic transfers, retention curves and conductivity properties, but do not take the mechanical behaviour of rock into account. Continuum damage models are generally applicable to dry materials. The few formulations for unsaturated media generally resort to Biot's representation of stress, which allows the decoupling of the mechanical and hydraulic behaviours in the constitutive equations. This theoretical approach cannot represent the effect of damage on suction rigidity. It is thus necessary to focus on the choice of stress-state variables.

The bases for a new damage model, formulated in net stress and suction independent state variables, are presented. This "HHMD"[1] model involves couplings between stress, pore pressures and damage, both in the behaviour law and in the transfer equations. It is aimed at representing the degradation of brittle unsaturated rock clays. Numerical programming and testing are currently being done on Θ-Stock finite element software (Gatmiri et al., 1999 ; Gatmiri and Hoor, 2007).

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[1] This is not an acronym. It is the name of the new model presented in the article: HH for the influence of both fluids (unsaturated conditions), M for mechanical effects, D for damage.

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